



Growth

Authors : Bill Davis, Horacio Porta and Jerry Uhl Producer : Bruce Carpenter

Publisher : [Math Everywhere, Inc.](http://MathEverywhere.com) Distributor & Translator: MathMonkeys, LLC

Table of Contents

PRINTED VOLUME 1: Modules 1.01 - 1.05

PRINTED VOLUME 2: Modules 1.06 - 1.09

PRINTED VOLUME 1: 1.01 - 1.05

1.01 - Growth

Basics

B.1) Growth of line functions $f(x) = a x + b$

B.2) Growth of power functions $f(x) = a x^k$

B.3) Growth of exponential functions $f(x) = a e^{r x}$

B.4) Dominance in the global scale

B.5) Percentage growth rate and dominance in the global scale

Tutorials

T.1) Global scale

T.2) Linear models

T.3) Data analysis and compromise lines

T.4) Functions given by data lists: Interpolation and analysis

T.5) The trig functions $\sin(x)$ and $\cos(x)$

T.6) Another linear model: Drinking and driving

Give It a Try

G.1) Line fundamentals

G.2) Global scale

G.3) Linear models

G.4) Compromise lines through data

G.5) Green globs

G.6) Percentage growth

G.7) Functions given by data lists: Interpolation and analysis

G.8) Another linear model: Drinking and driving

G.9) Interpolation and approximation*

Literacy Sheet

1.02 - Natural Logs and Exponentials

Basics

B.1) The natural base e and the natural logarithm

B.2) Percentage growth of exponential functions:
Doubling time and half life

B.3) Unnatural bases

Tutorials

T.1) Exponential models

T.2) Exponential data

T.3) e and Finance

Give It a Try

G.1) Exponential growth

G.2) Steady growth versus steady percentage growth

G.3) Exponential models

G.4) Exponential data analysis

G.5) Your money

G.6) Compounding every instant

G.7) Law and order

G.8) Unnatural bases

G.9) Reflecting patterns and wandering points

Literacy Sheet

1.03 - Instantaneous Growth Rates

Basics

B.1) Instantaneous growth rates

B.2) Instantaneous growth rate of x^k is measured by kx^{k-1}

B.3)

The instantaneous growth rate of $\sin(x)$ is measured by $\cos(x)$.

The instantaneous growth rate of $\cos(x)$ is measured by $-\sin(x)$.

B.4) The instantaneous growth rate of $\ln(x)$ is measured by $\frac{1}{x}$.

The instantaneous growth rate of e^x is measured by e^x .

This is why $\ln(x)$ is called the natural logarithm, and

why e is the natural base for exponentials.

Tutorials

T.1) Average growth rate versus instantaneous growth rate

T.2) Using the instantaneous growth rate $f'(x)$ to predict the plot of $f(x)$

T.3) Spread of disease

🗣️ T.4) Instantaneous growth rates in context

🗣️ Give It a Try

🗣️ G.1) Relating $f(x)$ and $f'(x)$

🗣️ G.2) Explaining LiveMath output

🗣️ G.3) Approximation of the instantaneous growth rate $f'(x)$ by

average growth rates $\frac{f(x+h) - f(x)}{h}$

🗣️ G.4) Using the instantaneous growth rate $f'(x)$ to predict the plot of $f(x)$

🗣️ G.5) Graphics action

🗣️ G.6) Up and down, maximum and minimum

🗣️ G.7) Spread of disease

🗣️ G.8) Average growth rate versus instantaneous growth rate

🗣️ G.9) Why folks study the instantaneous growth rate instead of instantaneous growth

🗣️ Literacy Sheet

🗣️ 1.04 - Rules

🗣️ Basics

🗣️ B.1) Derivatives, instantaneous growth rates, $f'(x)$ and $\frac{d}{dx}(f(x))$

🗣️ B.2) The chain rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) g'(x)$$

🗣️ B.3) General rules for taking derivatives

$$\frac{d}{dx}(f(x) + g(x), x) = f'(x) + g'(x)$$

$$\frac{d}{dx}(c f(x), x) = c f'(x)$$

$$\frac{d}{dx}(f(x) g(x), x) = f'(x) g(x) + f(x) g'(x)$$

🗣️ B.4) Using the logarithm to calculational advantage

🗣️ B.5) The instantaneous percentage growth rate of a positive function $f(x)$

is $100 \frac{f'(x)}{f(x)}$

🗣️ B.6) Exponential growth dominates power growth and power growth dominates logarithmic growth

🗣️ Tutorials

🗣️ T.1) Practicing with the chain rule

🗣️ T.2) Practicing with the chain rule, the product rule, and the power rule

🗣️ T.3) Linear dimension: length, area, volume and weight

Give It a Try

- G.1) Practicing with the chain rule
- G.2) Practicing with the chain rule, the product rule, and the power rule
- G.3) Global scale
- G.4) Exponential functions and their constant percentage growth rate
- G.5) Relating the plots of $f(x)$ and $f'(x)$
- G.6) $100\ln(f(x))$ and the instantaneous percentage growth rate
- G.7) Linear dimension: Length, area, volume, and weight
- G.8) Interest compounded every instant versus interest compounded every month

Literacy Sheet

1.05 - Using the Tools

Basics

- B.1) Using the derivative for finding maximum values and minimum values
- B.2) Using the derivative to help to get a good representative plot
- B.3) Using the derivative to fit data by curves: Line fit and Sine and Cosine wave fit

Tutorials

- T.1) Highest and lowest points on the graph
- T.2) Approximations by polynomials; Approximations by Sine and Cosine waves
- T.3) Fish gotta swim: The least energy
- T.4) Designing a box
- T.5) Largest and smallest

Give It a Try

- G.1) Good representative plots
- G.2) Highest and lowest points on the graph
- G.3) Approximations by polynomials and approximations by Sine and Cosine waves.
- G.4) Oil slicks
- G.5) The second derivative, $f''(x)$
- G.6) Driving the big Mack trucks
- G.7) The space shuttle Challenger and its O-rings
- G.8) Management analysis
- G.9) Up then down for $\frac{x^t}{e^x}$
- G.10) Other max-min problems
- G.11) At what age is the Bernese Mountain Dog growing the fastest?

Literacy Sheet

🔊 **PRINTED VOLUME 2: 1.06 - 1.09**

🔊 **1.06 - The Differential Equations of Calculus**

🔊 **Basics**

- 🔊 **B.1) The most important of all differential equations:**

$$y'(x) = r y(x)$$

and why you already know how to solve it

- 🔊 **B.2) The logistic differential equation**

$$y'(x) = r y(x) \left(1 - \frac{y(x)}{b} \right)$$

and how you get a formula for its solution

- 🔊 **B.3) Logistic growth is controlled growth**

- 🔊 **B.4) The differential equation**

$$y'(x) = r y(x) + b$$

and how to get a formula for its solution

🔊 **Tutorials**

- 🔊 **T.1) Radioactive decay and carbon dating**
🔊 **T.2) Socking money away**
🔊 **T.3) Wal-Mart: Exponential or logistic growth?**
🔊 **T.4) Pollution elimination**

🔊 **Give It a Try**

- 🔊 **G.1) Quick calculations**
🔊 **G.2) Data analysis**
🔊 **G.3) Logistic growth versus exponential growth**
🔊 **G.4) Why do they turn out this way?**
🔊 **G.5) Other differential equations**
🔊 **G.6) Managing your money**
🔊 **G.7) Which animals grow faster after their birth than they are growing at the time of their birth?**
🔊 **G.8) Newton's law of cooling: How a differential equation can help you enjoy your favorite cooled beverage**
🔊 **G.9) Pressure altimeters**

🔊 **Literacy Sheet**

🔊 **1.07 - The Race Track Principle**

🔊 **Basics**

- 🔊 **B.1) The Race Track Principle**
🔊 **B.2) The Race Track Principle and differential equations**
🔊 **B.3) The Race Track Principle and Euler's method of faking the plot of the solution of a differential equation**
🔊 **B.4) Tangent lines and the Race Track Principle**

🔊 **Tutorials**

- ☞ T.1) Using Euler's method to fake the plot of $f(x)$ given $f'(x)$ and one value of $f(x)$
- ☞ T.2) Using the Race Track Principle to help to estimate roundoff error
- ☞ T.3) If $f''(x)$ is always positive then tangent lines run below the curve

☞ Give It a Try

- ☞ G.1) Versions of the Race Track Principle
- ☞ G.2) Running Euler's faker
- ☞ G.3) The Race Track Principle and differential equations
- ☞ G.4) The error function $\text{Erf}(x)$
- ☞ G.5) Round off.
- ☞ G.6) Calculating accurate values of $\ln(x)$
- ☞ G.7) Calculating accurate values of e^x
- ☞ G.8) Euler's faker and the second derivative
- ☞ G.9) Inequalities
- ☞ G.10) The Law of the Mean
- ☞ G.11) If $f''(x)$ is never positive then tangent lines run above the curve; At points of inflection, the tangent line crosses the curve

☞ Literacy Sheet

☞ 1.08 - More Differential Equations

☞ Basics

- ☞ B.1) Euler's faker and LiveMath's Runge-Kutta faker
- ☞ B.2) Simultaneous differential equations: The predator-prey model

☞ Tutorials

- ☞ T.1) Using a differential equation to analyze Bubba's toot
- ☞ T.2) Analysis of the predator-prey model

☞ Give It a Try

- ☞ G.1) Variable interest rates
- ☞ G.2) Drinking and driving
- ☞ G.3) Further analysis of the predator-prey model
- ☞ G.4) The drug equation
- ☞ G.5) War games
- ☞ G.6) Logistic harvesting
- ☞ G.7) The logistic predator-prey model
- ☞ G.8) Epidemics
- ☞ G.9) Hints of chaos

☞ Literacy Sheet

☞ 1.09 - Parametric Plotting

☞ Basics

- ☞ B.1) Parametric plots in two dimensions: Circular parameters
- ☞ B.2) Parametric plots of curves in three dimensions
- ☞ B.3) Parametric plots of surfaces in three dimensions
- ☞ B.4) Derivatives for curves given parametrically : The cycloid

☞ Tutorials

- ☞ T.1) Parametric plotting for projectile motion
- ☞ T.2) Parametric plotting for designing a cam
- ☞ T.3) Parametric plotting of the predator-prey model
- ☞ T.4) Quick calculations

☞ Give It a Try

- ☞ G.1) Quick calculations
- ☞ G.2) Parametric plotting of circles and ellipses in two dimensions
- ☞ G.3) Elliptical orbits of planets and asteroids
- ☞ G.4) Parametric plotting of circles, tubes and horns in three dimensions
- ☞ G.5) Surfaces you can make by rotating curves
- ☞ G.6) Projectile marksmanship
- ☞ G.7) More cams
- ☞ G.8) Parametric plotting of a predator-prey model in which the prey don't reproduce and the predators don't die
- ☞ G.9) Politics and the environment
- ☞ G.10) Epidemics
- ☞ G.11) Collision?

☞ Literacy Sheet

☞ 1.10 - Holes, Jumps, and Asymptotes

☞ Basics

- ☞ B.1) It's Broken
- ☞ B.2) "At" vs. "Approaching"
- ☞ B.3) Limits
- ☞ B.4) Continuity

☞ Tutorials

- ☞ T.1) Limits
- ☞ T.2) Limit Rules
- ☞ T.3) Continuity

☞ Give It a Try

- ☞ G.1) Limits
- ☞ G.2) More Limits
- ☞ G.3) Continuity

☞ Literacy Sheet